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S-wave π^0 production In pp Collision In A Covariant OBE Model

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Abstract

The total cross section for the $pp \to pp\pi^0$ reaction at energies close to threshold is calculated in a covariant one-boson-exchange model. The amplitudes for the elementary $BN \to N\pi^0$ processes are taken to be the sum of s, u and t pole terms. The main contributions to the primary production amplitude is due to a σ meson exchange, which is strongly enhanced due to the t pole term. The NN and πN final state interactions are included coherently. The effects due to the πp interactions in pure isospin $\frac{1}{2}$ and $\frac{3}{2}$ channels are sizable but when taken in the appropriate isospin combination almost cancel out. Both the scale and energy dependence of the cross section are perfectly reproduced.

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In a recent study of the $pp \rightarrow pp\pi^0$ reaction at energies close to threshold, it is found that the angular distributions of the outgoing particles are isotropic, in agreement with the assumption that the reaction proceeds as a ${}^{33}P_0 \rightarrow {}^{31}S_0 s_0$ transition[1, 2]. Further more, the energy dependence of the total cross section, particle spectra and angular distributions are well reproduced by taking into account the pp final state interactions (FSI) only. The πN interactions seem to play a minor role, in marked difference with the similar $pp \rightarrow pp\eta$ reaction, where the ηN force is found to be essential for explaining the energy dependence of the cross section [3, 4]. Several model calculations[5, 7, 8] of S-wave pion production, which are based on a single nucleon and a pion rescattering mechanism, under estimate the cross section by a factor of 3-5. Inspired by a study of β-decay in nuclei, which indicates that the axial charge of the nuclear system is enhanced by heavy meson exchanges and, by using a simple operator form of the NN potential, Lee and Riska[9] have shown that meson exchange currents could explain the scale of the cross section. Horowitz et al.[10] performed similar calculations based on explicit one-boson-exchange (OBE) model for the NN interaction as well as for the evaluation of meson exchange contributions. The evaluation of these contributions depends on the virtuality of the meson exchanged and on how the on mass-shell amplitudes are extended into the off mass-shell region. There are several approach based on field theoretical models which allow such extension to be made but the results are model dependent. In the traditional phenomenological treatment[5, 6, 7, 8], the off mass-shell and on mass-shell amplitudes are of the same order of magnitude. Recently, several groups[11, 12, 13, 14] have concluded that the off mass-shell πN rescattering amplitude is enhanced with respect to the on mass-shell amplitude. Hernandez and Oset[11] by applying current algebra and PCAC constraints argue that this enhancement may bring the calculated cross section for the $pp \rightarrow pp\pi^0$ reaction into agreement with experiment. More detailed momentum-space calculations by Hanhart et al.[12] confirm this enhancement of the off mass-shell πN scattering amplitude but conclude that it is still too short to explain the scale of the cross section. Park et al.[13] and Cohen et al.[14] have applied a chiral perturbation theory, including chiral order 0 and 1 Lagrangian terms. They have shown that the off mass-shell πN scattering amplitude is enhanced considerably but has an opposite sign with respect to the on mass-shell amplitude. Because of this difference in sign, the rescattering term and the Born term contributing to the $pp \rightarrow pp\pi^0$ reaction interfere destructively, making the theoretical cross sections much smaller than experimental values. This enforces the importance of the heavy meson exchange contributions. Part of these contributions was included in Ref.[14].

In the present work we propose a covariant OBE model based on a two-step mechanism, where a virtual boson B (B=π , σ , η , ρ , ω ...) created on one of the incoming nucleons, is converted into a π^0 meson on a the second via a $BN \rightarrow \pi^0 N$ process (see diagram of Fig. 1). Similar to π^0 electroproduction amplitudes, the amplitudes for the elementary $BN \rightarrow N\pi^0$ processes are taken to be the sum of s, u and t pole terms. In fact only for the "effective" σ meson, a t pole term does contribute, giving rise to a strong enhancement of the cross section. With this model we obtain perfect agreement with the empirical cross-section data. Though differing in many details, the present work agrees with Horowitz et al.[10], that the σ meson exchange plays

an important role in pion production near threshold. Our model however is a natural extension of the relativistic OBE model used to analyze NN scattering data[15], where a single nucleon mechanism is not possible and all contributions correspond to completely connected diagrams. In the present formulation of the model, we may use any form of the $BN \to \pi^0 N$ amplitude so that comparison with other model calculations can be performed directly.

We use the following Lagrangian interaction:

$$\begin{aligned} L &= ig_{\pi NN}\bar{N}\gamma^5\tau N\pi + ig_{\eta NN}\bar{N}\gamma^5 N\eta + g_{\sigma NN}\bar{N}N\sigma + \\ & \bar{N}\left(g_{\rho NN}\gamma^\mu + i\frac{f_{\rho NN}}{2M}\sigma^{\mu\nu}q_\nu\right)\tau N\rho_\mu + g_{\omega NN}\bar{N}\gamma^\mu N\omega_\mu + g_{\delta NN}\bar{N}\tau N\delta \quad , \end{aligned} \tag{1}$$

with obvious notation. The coupling constants and the meson masses are taken from a fit to NN scattering data and are listed in table 5 of Ref.[15]. Using pseudovector couplings for the pseudoscalar mesons in the expression above would not alter the principal conclusions from the present work.

Following Refs.[3, 4] we write the transition amplitude in the form,

$$T_{23} \approx M_{23}^{(in)} \; T_{33}^{(el)} \quad , \tag{2}$$

where $M_{23}^{(in)}$ is the primary production amplitude which describes the transition from a two-nucleon state to a three-body state of two nucleon and a neutral pion, and $T_{33}^{(el)}$ is a FSI correction factor taken to be the on mass-shell elastic scattering amplitude of the $pp\pi^0 \to pp\pi^0$ transition. The validity of this approximation for π and η meson production in NN collisions is discussed in length in Refs.[3, 4]. Here we note that : (1) all inelastic interactions (and hence the coupling to other channels) are included into $M^{(in)}$ while $T_{33}^{(el)}$ depends on elastic interactions among the reaction products only, (2) as in the coherent formalism for three-body processes[16, 17], the different two-body interactions in the final state contribute coherently and (3) the transition amplitude has an overall phase factor identical to that of $T_{33}^{(el)}$ as required by general quantum mechanics constraints[16].

To calculate the transition amplitude we apply covariant perturbation theory techniques. We assume that the reaction is dominated by the mechanism depicted in Fig. 1 and write $M^{(in)}$ in the form,

$$M^{(in)} = \sum_B \left[T_{BN\to\pi^0 N}(p_4, k; p_2, q) G_B(q) S_{BNN}(p_3, p_1)\right] + [1 \leftrightarrow 2; 3 \leftrightarrow 4] \quad , \tag{3}$$

where p_i, q and k are 4-momenta of the ith nucleon, the exchanged boson and the outgoing pion. The sum runs over all possible B boson exchanges that may contribute to the process. The bracket $[1 \leftrightarrow 2; 3 \leftrightarrow 4]$ stands for a similar sum with the p_1, p_3 and p_2, p_4 momenta interchanged. In Eqn. 3, $T_{BN\to\pi^0 N}$ is the amplitude for the $BN \to \pi^0 N$ transition, $G_B(q)$ and $S_{BNN}(p_3, p_1)$ are the propagator and source function of the meson exchanged, respectively. By using covariant meson propagators and form factors and covariant parametrizations for S_{BNN} and $T_{BN\to\pi^0 N}$, the transition

amplitude is expressed in terms of invariant functions. For a scalar and pseudoscalar meson exchanges,

$$S_{SNN}(p_1,p_3) = \bar{u}(p_3)\ I\ u(p_1)\ F_S(q)\ \ ; S_{PNN}(p_1,p_3) = \bar{u}(p_3)\ \gamma^5\ I\ u(p_1)\ F_P(q)\ \ , \quad (4)$$

where $F_B(q)$ is a source form factor and I the appropriate isospin operator. Here u is a nucleon Dirac spinor and $p_3 = p_1 - q$ is the final nucleon momentum. A most general vector source contains vector and tensor current terms,

$$S^{\mu}_{VNN}(p_1,p_3) = \bar{u}(p_3)\left[\gamma^{\mu}F_V^{(1)}(q_{13}^2) + i\sigma^{\mu\nu}q_{\nu}F_V^{(2)}(q_{13}^2)\right] I u(p_1)\ \ , \quad (5)$$

with the vector source form factors, $F_V^{(i)}$, being analogous to the electromagnetic form factors of the nucleon. In the calculations to be presented below, all source form factors are taken in their lowest order perturbative approximation. The expression, Eqn. 5, contains conserved currents only and therefore satisfies current conservation.

The amplitude for a scalar meson-nucleon scattering, $SN \rightarrow PN$, is written in the usual form[18],

$$T_{SN\rightarrow P_2 N}(p_4,k;p_2,q) = \bar{u}(p_4)\gamma^5\ \left[A + \frac{1}{2}(\not{k} + \not{q})B\right] u(p_2)\ \ , \quad (6)$$

where A and B are functions of the Mandelstam variables, having the same isospin structure. To obtain the amplitude corresponding to a to a pseudoscalar meson scattering, $P_1 N \rightarrow P_2 N$, one has to remove the γ^5 in the expression above.

Similarly, for a vector meson exchange , the amplitude for the $VN \rightarrow \pi^0 N$ transition is,

$$T^{\mu}_{VN\rightarrow\pi^0 N}(p_4,k;p_2,q) = \bar{u}(p_4)\gamma^5\left[\gamma^{\mu}\mathcal{A}_{12} + p_4^{\mu}\mathcal{A}_{34} + k^{\mu}\mathcal{A}_{56} + q^{\mu}\mathcal{A}_{78}\right] u(p_2)\ \ , \quad (7)$$

where $\mathcal{A}_{ij}$ stands for the combination $(A_i\ +\ \not{k}A_j)$.

In the analysis to be presented below, we include π , σ , η , ρ and ω meson exchanges. In order to calculate these contributions to the transition amplitude for the $pp \rightarrow pp\pi^0$ reaction we call attention to certain kinematical features of this process.

One important aspect of the mechanism of Fig. 1 is that at the π^0 production threshold, the transferred 4-momentum is space-like, $q^2 = -3.3 fm^{-2}$. This is very much the same kinematics that occurs in π^0 electroproduction through vector meson exchanges, suggesting that the half off mass-shell amplitudes $T_{BN\rightarrow\pi^0 N}$ which appear in Eqn. 3, can be calculated using the usual calculation procedure of electroproduction amplitudes[19]. Particularly, each of the amplitudes $T_{\eta N\rightarrow\pi^0 N}$, $T_{\sigma N\rightarrow\pi^0 N}$, $T_{\rho N\rightarrow\pi^0 N}$ and $T_{\omega N\rightarrow\pi^0 N}$ are far below the physical region and near the relevant poles, and therefore can be approximated as the sum of s, u and t pole terms. In fact, a t pole contributes to the $T_{\sigma N\rightarrow\pi^0 N}$ only and as shown bellow it dominates the reaction amplitude.

The situation is drastically different for π exchange because the $T_{\pi^0 N\rightarrow\pi^0 N}$ is in the physical region close to threshold. It is now well established that the on mass-shell amplitude for the $\pi^0 p \rightarrow \pi^0 p$ scattering is very small and in the soft pion limit is equal to zero[21] near threshold. Based on general quantum mechanical constraints[20, 16],

the half off mass-shell πN scattering amplitude can be written as the product of the on mass-shell amplitude and a real Kowalski-Noyes function[20], approaching unity in the limit of $q^2 = m_\pi^2$. The derivation of this function is difficult and model dependent. In the traditional treatment of offshellness, this function corresponds to the form factor of the B meson leg (see diagram 1), which slightly enhances the off mass-shell amplitude with respect to the on mass-shell amplitude. Hernandez and Oset[11] suggested two models that yield rather large enhancement. More detailed studies[12, 13, 14] , agree that the enhancement due to offshellness, though differing in sign, is significant and must be taken into account.

In order to study the influence of π exchange we evaluate the $pp \to pp\pi^0$ cross section by (1) neglecting π exchange (this correspond to an exact cancellation between the direct and rescattering term of Refs.[13, 14]), and (2) using the enhanced πN amplitude of Park et al.[13]. In agreement with Refs.[12, 13, 14], it is found that, using the off mass-shell rescattering amplitude rather than the on mass-shell amplitude, influences the calculated cross section for the $pp \to pp\pi^0$ rather little.

As noted already, a significant contribution to the $T_{\sigma N \to \pi^0 N}$ amplitude is due to the t pole term. To calculate this require knowledge of the $g_{\sigma\pi\pi}$ coupling constant. Taking the Lagrangian density as $L = g_{\sigma\pi\pi} M\pi \cdot \pi\sigma$ and by a simple one loop calculations this constant can be related to the $g_{\sigma NN}$ through,

$$\frac{g_{\sigma\pi\pi}^2}{4\pi} \approx \left(\frac{4\pi}{3}\right)^2 \frac{g_{\sigma NN}^2}{4\pi} \left(\ln\frac{M}{m_\pi} - 1\right)^{-2} \left(\frac{g_{\pi NN}^2}{4\pi}\right)^{-2} + O\left(\frac{m_\pi^2}{M^2}\right) \quad , \tag{8}$$

where m_π and M are the pion and proton mass. With the constants $g_{\sigma NN}^2/4\pi = 8.28$ and $g_{\pi NN}^2/4\pi = 14.6$ one obtains $g_{\sigma\pi\pi} = (\ 3.3 \pm 0.1\ )$. The errors is due to neglecting terms of the order $O\left(\frac{m_\pi^2}{M^2}\right)$. The transition amplitude of the reaction (solid curve) and the different meson exchange contributions are drawn in Fig. 2. The σ meson exchange amplitude exceeds by far any other contribution. The relative phases for σ , ρ , ω exchanges are predicted to be +1 and add constructively. The η exchange amplitude has a negative phase and interfere destructively, scaling down the transition amplitude to the proper magnitude required to explain the data.

Finally, the cross section is calculated from the expression,

$$\sigma_T = \frac{M^4}{16(2\pi)^5\sqrt{(s)}\mathbf{p}_1} \int \frac{d^3\mathbf{p}_3}{E_3}\ \frac{d^3\mathbf{p}_4}{E_4}\ \frac{d^3\mathbf{p}_\eta}{E_\eta}\ |\ Z\ |^2 Sp\left(M^{(in)}\ M^{(in)\dagger}\right) \delta^4(p_i - p_f), \tag{9}$$

where $Sp\left(M^{(in)}\ M^{(in)\dagger}\right)$ denotes the trace over spinor states, and Z is the three-body FSI correction factor of Refs.[3, 4]. In the analysis presented below this factor is estimated from πN and NN elastic S-wave scattering phase shifts[3, 4]. The S-wave NN phase shift is obtained from the effective range expansion which includes Coulomb interaction between the two protons. We have used the scattering length $a_{pp} = -7.82$ fm and an effective range $r_{pp} = 2.7$ fm of Ref.[23]. The S11 and S13 πN scattering lengths are taken to be $a_1 = 0.173\ m_\pi^{-1}$ and $a_3 = -0.101\ m_\pi^{-1}$, respectively[24].

Our predictions for the total cross section are drawn in Figs. 3 along with the available $pp \to pp\pi^0$ data. The measured cross-section is reproduced remarkably well

with the π exchange contribution taken to be zero (solid curve). Note that all of the model parameters are determined independently from NN and πN scattering data and that none of these parameters have been adjusted to the $pp \rightarrow pp\pi^0$ reaction. Our predictions with the off mass-shell amplitude $T_{\pi^0 p \rightarrow \pi^0 p}$ of Ref.[13] are drawn as a dashed curve. As the relative phase of the π exchange contribution and the other meson exchanges is not known we have repeated these calculations with the $T_{\pi^0 p \rightarrow \pi^0 p}$ taken with a reversed sign (small dashed line). Thus the effects of the π virtuality vary the calculated cross section by $\approx 15\%$ only, and as in Refs.[12, 13, 14] we may conclude that offshellness of the exchanged π would not resolve the discrepancy with experiment.

In agreement with the model of Horowitz et al.[10], the main contribution is due to σ exchange with ratios $M_\sigma : M_\omega : M_\eta : M_\rho \approx 100 : 40 : 8 : 7$. Although the contribution of the η meson is rather weak it becomes effective through interference terms with the other exchange contributions. It is interesting to note that for the σ exchange, the sum of the s and u pole terms alone amounts to $\approx 6\%$ of the η amplitude only, and that the main contribution is due to the t pole term which was not included in previous studies. The contribution from the δ meson exchange is extremely weak for two reasons. The first is that the s and u pole terms have about the same magnitude but opposite signs. The second is that the mass of the δ meson is high and the coupling constant $g_{\delta NN}$ is small. All these scale the δ exchange contribution to less than 0.7% of the η exchange contribution.

The transition amplitude is practically constant near threshold and the energy dependence is determined almost solely by the FSI correction factor and phase space. An important property of the coherent formalism is that the different two body interactions among the out going particles contribute coherently. Although the meson-nucleon interactions are weak with respect to the NN interaction, they become influential through interference. To see this, we draw in Fig. 4 the cross section corrected for FSI interactions assuming pure I=$\frac{1}{2}$ and I=$\frac{3}{2}$ interactions for the outgoing π^0N pairs. The energy dependence of the cross section differ significantly for the two channels. But, when the πN interactions are taken in the proper isospin combination, their overall contribution to the FSI factor almost cancels out, leading to an energy dependence practically identical with the one corrected for the pp FSI (large dash curve) only.

In summary, we have used OBE model to calculate S-wave production cross section for the $pp \rightarrow pp\pi^0$ reaction by assuming a two-step mechanism where, a boson formed on one of the nucleons converts into a π on the other through the $BN \rightarrow \pi^0 N$ process. The kinematic conditions of the $pp \rightarrow pp\pi^0$ reaction at threshold, force the $T_{BN \rightarrow \pi^0 N}$ amplitudes for the heavy mesons being far below the physical region, and hence could be estimated as the sum of s, u and t pole terms. It is found that the t pole term of σ meson exchange is essential in order to reproduce the scale of the cross section. Using off mass-shell πN scattering amplitude as obtained from chiral perturbation theory would not affect these conclusions.

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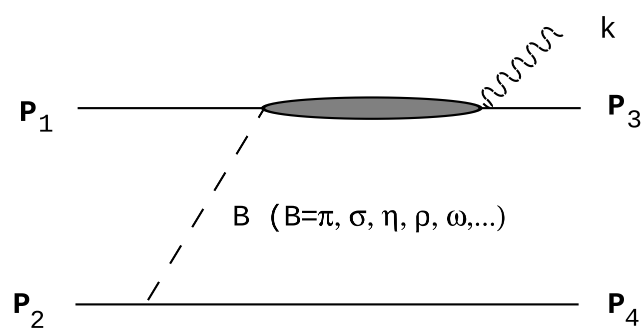

Figure 1: The primary production mechanism for the $NN \rightarrow NN\pi^0$ reaction.

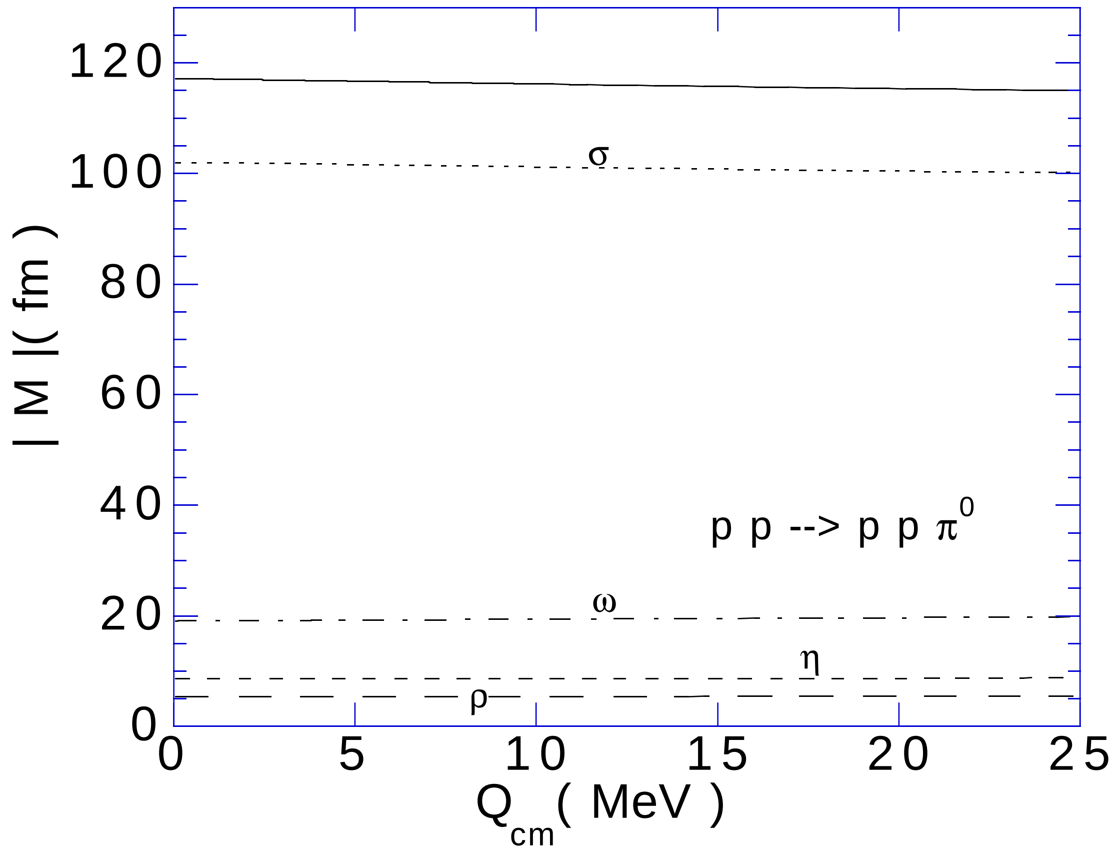

Figure 2: Predictions for the different meson exchange amplitudes to the $pp \rightarrow pp\pi^0$ reaction vs. Q_{cm}, the energy available in the center of mass system. The primary production amplitude is drawn as a solid curve. Note that the relative phases are taken into account.

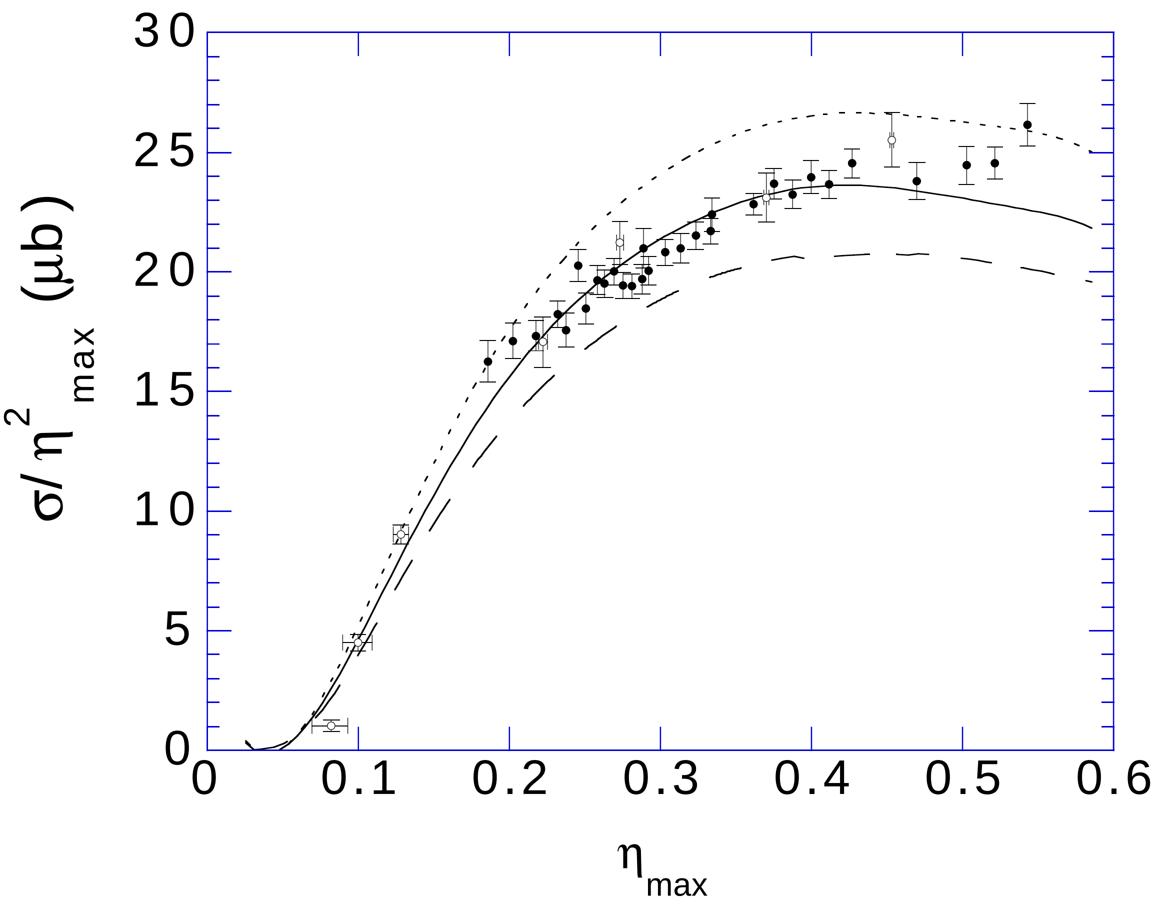

Figure 3: Predictions for the total cross section of the $pp \to pp\pi^0$ reaction. Predictions with the π exchange scattering amplitude taken to be zero are drawn as a solid curve. Those with the $T_{\pi^0 p \to \pi^0 p}$ amplitude taken from Park et al.[13] are drawn as a dashed line. The small dashed curve gives predictions with the amplitude of Park et al.[13] taken with the sign reversed. The data points are taken from Refs.[1, 2]

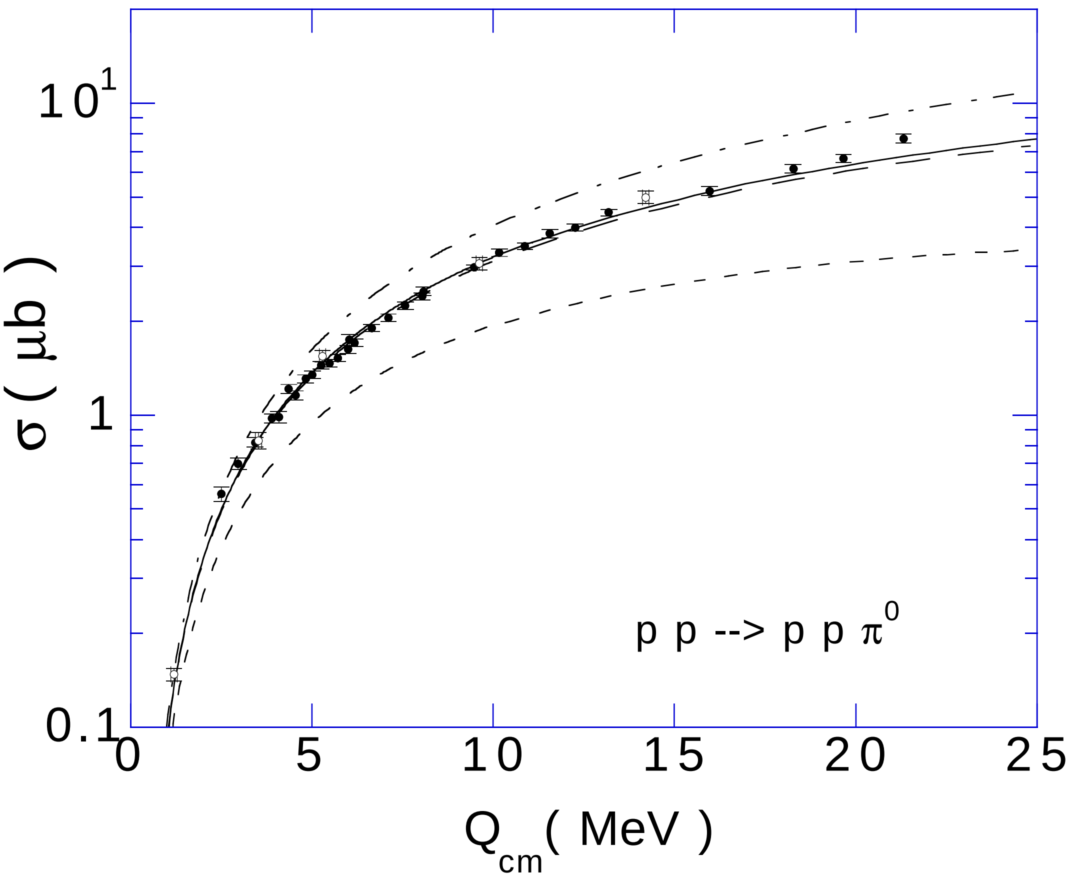

Figure 4: FSI corrections in pure isospin πN channels. Integrated energy cross sections calculated with the assumption that the interacting πN pair is scattered in isospin I=$\frac{1}{2}$ (small dashed curve) and I =$\frac{3}{2}$ (dot-dashed curve). The solid line is that obtained with the π N interactions taken in the appropriate isospin combinations. Predictions which account for the pp FSI only are (large dashed curve) are nearly identical with the solid curve.